

2.7. Derivatives and Rates of Change:

1. Watch the video at:

<https://www.khanacademy.org/math/differential-calculus/derivative-intro-dc/intro-to-diff-calculus-dc/v/newton-leibniz-and-usain-bolt>

2. Solve 10 exercises from:

<https://www.khanacademy.org/math/differential-calculus/derivative-intro-dc/intro-to-diff-calculus-dc/e/graphs-of-functions-and-their-derivatives>

3. Write an expression for the slope of the tangent line to the curve $y = f(x)$ at the point $(a, f(a))$.

4. Write a formula for $f'(a)$ for a generic function $f(x)$. Can you write another formula for $f'(a)$?

4. Find $f'(a)$ for:

a) $f(x) = 3x^2 - 4x + 1$

b) $f(x) = 2x^3 + x$

c) $f(x) = x^{-2}$

d) $f(x) = \frac{2x+1}{x+3}$

5. Find the slope of the tangent line to the parabola $y = 4x - x^2$ at the point $(1, 3)$. Find an equation of this tangent line.

6. Find the slope of the tangent line to the curve $y = x - x^3$ at the point $(1, 0)$. Find an equation of this tangent line.

7. Find an equation of the tangent line to the curve $y = \frac{2x+1}{x+2}$ at the point $(1, 1)$.

8. a) Find the slope of the tangent line to the curve $y = 3 + 4x^2 - 2x^3$ at the point where $x = a$.

b) Using part a), find equations of the tangent lines at the points (1,5) and (2,3).

c) Using a graphical device, graph the curve and both tangents on a common screen.

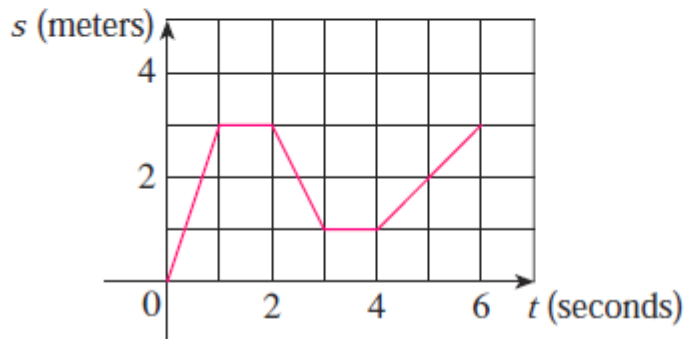
9. Suppose that an object moves along a straight line with position $f(t)$ at time t . Write an expression for the instantaneous velocity of the object at time $t = a$. How can you interpret this velocity in terms of the graph of $f(t)$?

10.

(a) A particle starts by moving to the right along a horizontal line; the graph of its position function is shown.

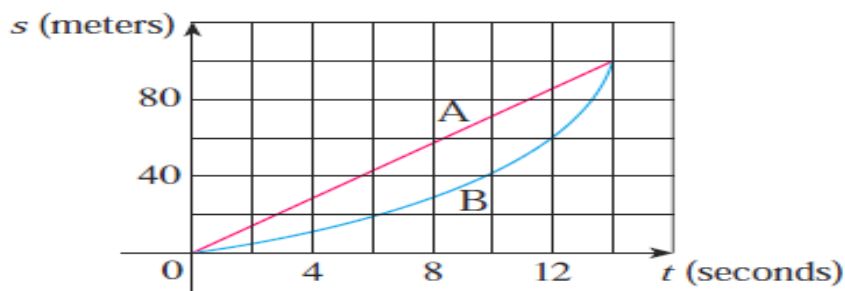
When is the particle moving to the right? Moving to the left? Standing still?

(b) Draw a graph of the velocity function.



11.

Shown are graphs of the position functions of two runners, A and B, who run a 100-m race and finish in a tie.



a) describe and compare how the runner run the race in terms of their instantaneous speeds.

b) at what time is the distance between the runners the greatest?

c) at what time do they have the same instantaneous speeds?

2.8. The derivative as a function:

1. Write a formula for the derivative $f'(x)$ of a function $f(x)$. Can you write another formula for $f'(x)$?

2. Calculate $f'(x)$ using your definition from exercise 1. State the domain of $f(x)$ and the domain of $f'(x)$ in each case:

a) $f(x) = \frac{1}{2}x - \frac{1}{3}$

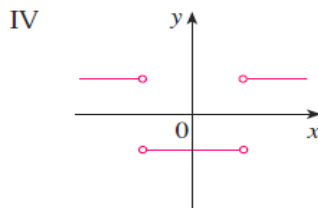
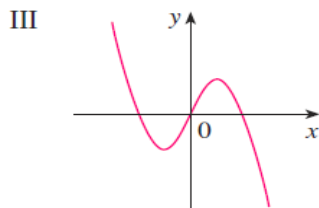
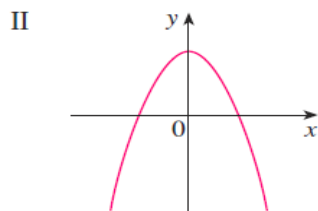
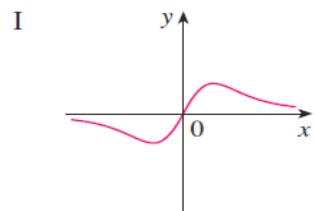
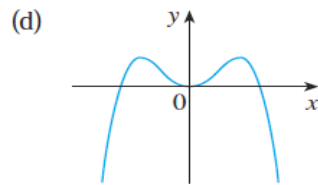
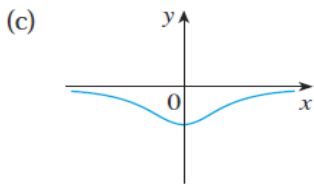
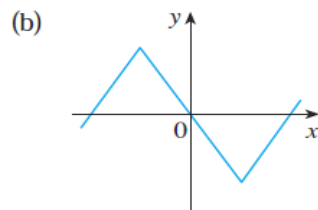
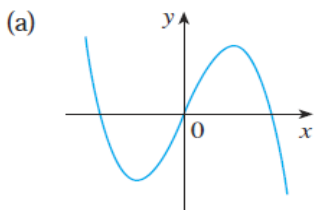
b) $f(x) = \sqrt{x}$

c) $f(x) = \frac{1}{\sqrt{x}}$

d) $y = \frac{1-2x}{3+x}$

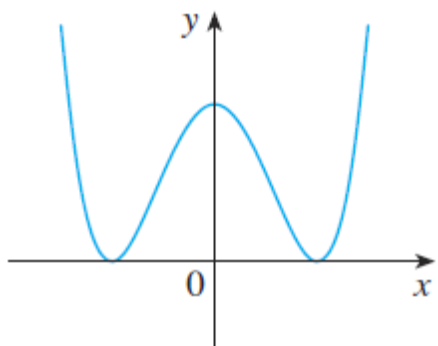
3.

Match the graph of each function in (a)–(d) with the graph of its derivative in I–IV. Give reasons for your choices.

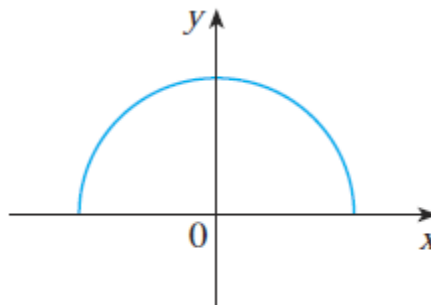


4. The graph of a function $f(x)$ is given. For each function, sketch the graph of its derivative $f'(x)$ directly underneath it:

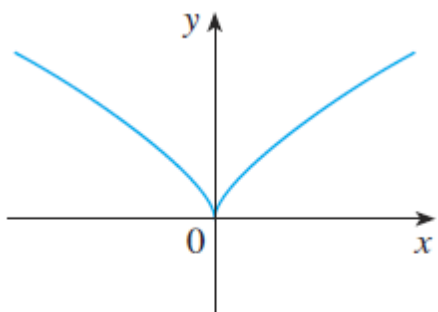
a)



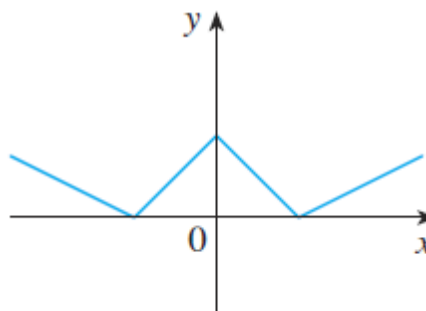
b)



c)



d)



5.

a) What does it mean for $f(x)$ to be differentiable at a ?

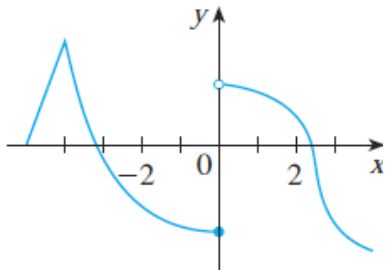
b) What is the relationship between differentiability and continuity for a function?

c) Sketch the graph of a function which is continuous but not differentiable at $a = 2$.

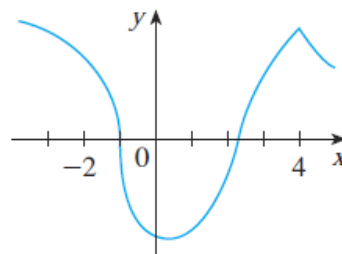
d) Describe several ways in which a function can fail to be differentiable. Illustrate each case with sketches.

7. The graph of $f(x)$ is given. In each case, state with reasons all points at which $f(x)$ is not differentiable:

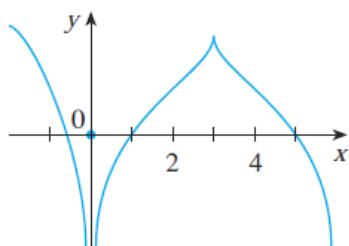
a)



b)



c)



d)

